AP-Calculus AB (2019-2020) MAST Academy

Ms. Avendano's Summer Assignment

In the attachments you will see Four different assignments that are review from Basic Algebra and Pre-Calculus which are requirements for AP Calculus AB . The assignments will be collected the first day of School 2019-2020 for a grade and will be a test on those assignments the first Thursday or Friday depending of your schedule.

After working on the assignments , if you see deficiencies on some topics, search for the topic on khan Academy to study them, you must be ready to do outstanding next school year.

Assign 1 : Simplifying expressions and Algebraic Gymnastics (33problems)

Assign 2: Calculus Pre-requisites (5-21)

Assign 3 : Pre-Calculus /Trigonometry (12 problems)

Assign 4: Chapter 1-Review (1-28)

Name_____ Date_____ Period_____

WS P.3—Simplifying Expressions and Algebraic Gymnastics

Show all work on notebook paper. No Calculator

1. Find the exact value of each expression

(a) $\log_{10} 25 + \log_{10} 4$ (b) $e^{4\ln 2}$

2. Solve each of the following equations for x. Find the simplified, exact value.

(a) $e^x = 3$ (b) $e^{e^x} = 3$ (c) $\log_3(x+1) = 2$ (d) $\log_3 27 = x$

Multiple Choice

_____ 3. Rationalize the numerator of $\dfrac{\sqrt{x+4}-\sqrt{x-2}}{x}$

(A) $\dfrac{2}{x\left(\sqrt{x+4}+\sqrt{x-2}\right)}$ (B) $\dfrac{6}{x\left(\sqrt{x+4}-\sqrt{x-2}\right)}$ (C) $\dfrac{6}{x\left(\sqrt{x+4}+\sqrt{x-2}\right)}$

(D) $\dfrac{2x}{\sqrt{x+4}+\sqrt{x-2}}$ (E) $\dfrac{6x}{\sqrt{x+4}-\sqrt{x-2}}$

_____ 4. Which, if any, of the following statements are true when a, b are real numbers?

I. For all positive a and b, $\sqrt{a+b}=\sqrt{a}+\sqrt{b}$.

II. For all a and b, $\sqrt{(a+b)^2}=|a+b|$.

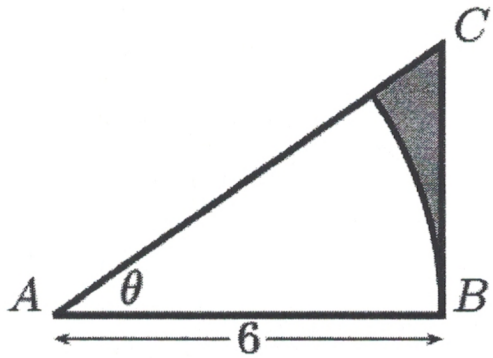
III. For all positive a and b, $\dfrac{a-b}{\sqrt{a}+\sqrt{b}}=\sqrt{a}+\sqrt{b}$.

(A) III only (B) all of them (C) I and II only (D) II only (E) II and III only
(F) none of them (G) I and III only (H) I only

_____ 5. Simplify the expression $\dfrac{1+\dfrac{2}{x-3}}{5+40\left(\dfrac{x}{x^2-9}\right)}$

(A) $\dfrac{1}{5}\left(\dfrac{x+3}{2x+9}\right)$ (B) $\dfrac{x+3}{x-9}$ (C) $\dfrac{1}{5}\left(\dfrac{x+3}{x+9}\right)$ (D) $\dfrac{x+3}{2x-9}$ (E) $\dfrac{1}{5}\left(\dfrac{x-3}{x+9}\right)$ (F) $\dfrac{x-3}{x-9}$

_____ 6. The shaded area in the figure is the complement of the sector of a circle of radius 6 inches lying inside the right triangle ΔABC with the angle θ being expressed in radians. Express this shaded area as a function of S, of θ.



(A) $S(\theta)=36(\tan\theta-\theta)$ (B) $S(\theta)=36(\sin\theta-\theta)$ (C) $S(\theta)=18(\sin\theta-\theta)$
(D) $S(\theta)=18(\cos\theta-\theta)$ (E) $S(\theta)=18(\tan\theta-\theta)$

_____ 7. Which of the following statements are true?

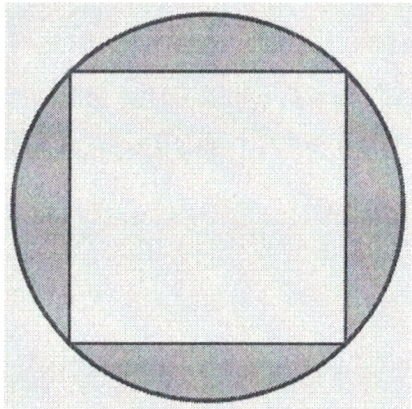
I. The circle $(x-1)^2+(y-2)^2=1$ has radius $=1$.

II. The circle $(x-5)^2+(y-6)^2=9$ has center $=(6,5)$.

III. The circle $(x-4)^2+(y-4)^2=25$ has y-intercepts $=1,\ 7$.

(A) I only (B) II only (C) I and III only (D) III only (E) II and III only
(F) none of them (G) all of them (H) I and II only

_____ 8. Find the area of the shaded region shown outside the square and inside the circle when the area of the circle is 25π sq. units.



(A) $5(4-\pi)$ sq. units (B) $5(\pi-1)$ sq. units (C) $25(\pi-2)$ sq. units
(D) $5(\pi-2)$ sq. units (E) $25(\pi-1)$ sq. units (F) $25(4-\pi)$ sq. units

_____ 9. Simplify the difference quotient $\frac{f(x+h)-f(x)}{h}$, $(h \neq 0)$, when $f(x)=2x^2-4x-4$.

(A) $4x+4+2h$ (B) $4x-4+2h$ (C) $2x+4+2h$ (D) $2x-4+2h$ (E) $4x-4$

_____ 10. Captain Calculus can leap over tall buildings. When he does so, his height s (in feet) off the ground after t seconds is given by $s(t)=-t^2+7t+34$. For how many seconds is Captain Calculus more that 40 feet off the ground?

(A) 6 sec (B) $\frac{9}{2}$ sec (C) $\frac{11}{2}$ sec (D) $\frac{5}{2}$ sec (E) 5 sec

_____ 11. If $f(x)=2x-1$ and $g(x)=x+3$, which of the following gives $(f \circ g)(2)$?

(A) 2 (B) 6 (C) 7 (D) 9 (E) 10

_____ 12. Which of the following is a solution of the equation $2-3^x=-1$?

(A) $x=-2$ (B) $x=-1$ (C) $x=0$ (D) $x=1$ (E) No solution

_____ 13. The length L of a rectangle is twice as long as its width W. Which of the following gives the area A of the rectangle as a function of its width?

(A) $A(W)=3W$ (B) $A(W)=\frac{1}{2}W^2$ (C) $A(W)=2W^2$

(D) $A(W)=W^2+2W$ (E) $A(W)=W^2-2W$

_____ 14. If $p(x)=(x+2)(x+k)$ and if the remainder is 12 when $p(x)$ is divided by $x-1$, then $k=$

(A) 2 (B) 3 (C) 6 (D) 11 (E) 13

_____ 15. The set of all points (e^t,t), where t is a real number, is the graph of $y=$

(A) $\frac{1}{e^x}$ (B) $e^{1/x}$ (C) $xe^{1/x}$ (D) $\frac{1}{\ln x}$ (E) $\ln x$

_____ 16. If $f(x)=\frac{4}{x-1}$ and $g(x)=2x$, then the solutions of $f(g(x))=g(f(x))$ is

(A) $\left\{\frac{1}{3}\right\}$ (B) $\{2\}$ (C) $\{3\}$ (D) $\{-1,2\}$ (E) $\left\{\frac{1}{3},2\right\}$

_____ 17. If the function f is defined by $f(x)=x^5-1$, then f^{-1}, the inverse function of f, is defined by $f^{-1}(x)=$

(A) $\frac{1}{\sqrt[5]{x}+1}$ (B) $\frac{1}{\sqrt[5]{x+1}}$ (C) $\sqrt[5]{x-1}$ (D) $\sqrt[5]{x}-1$ (E) $\sqrt[5]{x+1}$

_____ 18. If a, b, c, d, and e are real numbers and $a\neq 0$, then the polynomial equation $ax^7+bx^5+cx^3+dx+e=0$ has

(A) only one real root (B) at least one real root (C) an odd number of nonreal roots
(D) no real roots (E) no positive real roots

_____ 19. What are all values of k for which the graph of $y=x^3-3x^2+k$ will have three distinct x-intercepts?

(A) All $k>0$ (B) All $k<4$ (C) $k=0,4$ (D) $0<k<4$ (E) All k

_____ 20. If $f(g(x))=x^3+3x^2+4x+5$ and $g(x)=5$, then $g(f(x))=$

(A) $5x^2+15x+25$ (B) $5x^3+15x^2+20x+25$ (C) 1125 (D) 225 (E) 5

_____ 21. If $f(x)=2x^3+Ax^2+Bx-5$ and if $f(2)=3$ and $f(-2)=-37$, what is the value of $A+B$?

(A) -6 (B) -3 (C) -1 (D) 2 (E) It cannot be determined from the information given

_____ 22. Suppose that f is a function that is defined for all real numbers. Which of the following conditions assures that f has an inverse function?

(A) The function f is periodic (B) The function f is symmetric with respect to the y-axis
(C) The function f is concave up (D) The function f is a strictly increasing function
The function f is continuous

_____ 23. If $\log_a\left(2^a\right)=\frac{a}{4}$, then $a=$

(A) 2 (B) 4 (C) 8 (D) 16 (E) 32

_____ 24. If $f(g(x))=\ln\left(x^2+4\right)$, $f(x)=\ln\left(x^2\right)$, and $g(x)>0$ for all real x, then $g(x)=$

(A) $\frac{1}{\sqrt{x^2+4}}$ (B) $\frac{1}{x^2+4}$ (C) $\sqrt{x^2+4}$ (D) x^2+4 (E) $x+2$

_____ 25. If $\ln x - \ln\left(\frac{1}{x}\right) = 2$, then $x =$

(A) $\frac{1}{e^2}$ (B) $\frac{1}{e}$ (C) e (D) $2e$ (E) e^2

_____ 26. If $f(x) = \frac{x}{x+1}$, then the inverse function, f^{-1}, is given by $f^{-1}(x) =$

(A) $\frac{x-1}{x}$ (B) $\frac{x+1}{x}$ (C) $\frac{x}{1-x}$ (D) $\frac{x}{x+1}$ (E) x

_____ 27. If $f(x) = e^x \sin x$, then the number of zeros of f on the closed interval $[0, 2\pi]$ is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

_____ 28. If h is the function given by $h(x) = f(g(x))$, where $f(x) = 3x^2 - 1$ and $g(x) = |x|$, then $h(x) =$

(A) $3x^3 - |x|$ (B) $|3x^2 - 1|$ (C) $3x^2|x| - 1$ (D) $3|x| - 1$ (E) $3x^2 - 1$

_____ 29. If $e^{g(x)} = \dfrac{x^x}{x^2-1}$, then $g(x) =$

(A) $x\ln x - 2x$ (B) $\dfrac{\ln x}{2}$ (C) $(x-2)\ln x$ (D) $\dfrac{x\ln x}{\ln(x^2-1)}$ (E) $x\ln x - \ln(x^2-1)$

_____ 30. $\dfrac{\ln(x^3e^x)}{x} =$

(A) $\dfrac{3(\ln x + e^x)}{x}$ (B) $\ln(x^3e^x - x)$ (C) $\ln x^2 + 1$ (D) $\dfrac{3\ln x + x}{x}$ (E) $\dfrac{3\ln x}{x}$

_____ 31. If $f(g(x)) = \sec(x^3+4)$, $f(x) = \sec x^3$, and $g(x)$ is **not** an integer multiple of $\dfrac{\pi}{2}$, then $g(x) =$

(A) $\sqrt[3]{x+4}$ (B) $\sqrt[3]{x-4}$ (C) $\sqrt[3]{x^3+4}$ (D) $\sqrt[3]{x}-4$ (E) $\sqrt[3]{x}+4$

_____ 32. If $f(x) = \log_b x$, then $f(bx) =$

(A) $bf(x)$ (B) $f(b)f(x)$ (C) $1+f(x)$ (D) $xf(b)$ (E) $f(x)$

_____ 33. Which of the following statements is true?

(A) $\log_{\frac{1}{2}} 2 < \log_{\frac{1}{\sqrt{2}}} 2$ (B) $\log_3(2+4) = \log_3 2 + \log_3 4$ (C) $\log 2 > \log 4$

(D) $\log_{\frac{1}{5}}\left(5\sqrt{5}\right) = \frac{2}{3}$ (E) $\log_{\frac{1}{2}} 2 - \log_{\frac{1}{2}} 4 = \log_{\frac{1}{2}} 2$

Name_____ Date_____ Period_____

Calculus Prerequisites

Worksheet

Work the following on notebook paper. All work must be shown. Use your **calculator only on problems 14-17**

Find the equations of the asymptotes (horizontal, vertical, and slant), symmetry, and intercepts, then sketch the graph.

5. $y = \dfrac{2x^2 - 8}{x^2 - 16}$

6. $y = \dfrac{x^2 - 2x - 3}{x + 2}$

Solve.

7. $(x+1)(x-3)^3(x+2)^2 \geq 0$

8. $\dfrac{4x-3}{x+1} < 0$

Solve. Show all steps. Give decimal answers correct to **three** decimal places.

14. $e^{2x} - e^x - 12 = 0$

15. $\log_3(x+4) - \log_3(x-1) = 2$

16. $\log_2\left(\log_3\left(\log_5 x\right)\right) = 0$

17. The number of junior and senior students at NBHS infected with the "Math Bug" t days after exposure is modeled by the function $P(t)=\dfrac{500}{1+3^{3-t}}$.

    a. How many students were originally infected?
    b. How many of these students were infected after four days?
    c. When will 200 of these students be infected?
    d. What is the maximum number of students that will be infected?

Evaluate.

18. $\cos\left(Sin^{-1}\left(-\dfrac{\sqrt{3}}{2}\right)\right)$

19. $\tan\left(Arc\sec(3x)\right)$

---

20. Convert $r^2+6r\cos\theta=0$ into rectangular form.

21. Sketch the graph of $r=2+2\sin\theta$.

22. Sketch the graph of $r=1-2\cos\theta$.

Pre-Calculus / Trigonometry

73 31. If $\log_a\left(2^a\right)=\frac{a}{4}$, then $a=$

(A) 2 (B) 4 (C) 8 (D) 16 (E) 32

73 2. If $f(x)=x^3+3x^2+4x+5$ and $g(x)=5$, then $g(f(x))=$

(A) $5x^2+15x+25$ (B) $5x^3+15x^2+20x+25$ (C) 1125

(D) 225 (E) 5

88 2. What is the domain of the function f given by $f(x)=\frac{\sqrt{x^2-4}}{x-3}$?

(A) $\{x: x\neq 3\}$ (B) $\{x: |x|\leq 2\}$ (C) $\{x: |x|\geq 2\}$

(D) $\{x: |x|\geq 2 \text{ and } x\neq 3\}$ (E) $\{x: x\geq 2 \text{ and } x\neq 3\}$

88 32. Which of the following does NOT have a period of π ?

(A) $f(x)=\sin\left(\frac{1}{2}x\right)$ (B) $f(x)=|\sin x|$ (C) $f(x)=\sin^2 x$

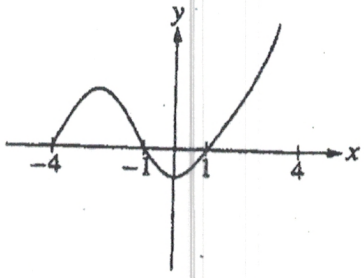
(D) $f(x)=\tan x$ (E) $f(x)=\tan^2 x$

73 5. If $f(x)=e^x$, which of the following lines is an asymptote to the graph of f?

(A) $y=0$ (B) $x=0$ (C) $y=x$ (D) $y=-x$ (E) $y=1$

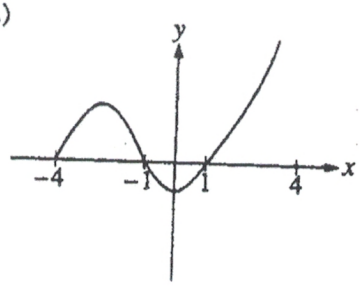
88 42. The graph of which of the following equations has $y=1$ as an asymptote?

(A) $y=\ln x$ (B) $y=\sin x$ (C) $y=\frac{x}{x+1}$ (D) $y=\frac{x^2}{x-1}$ (E) $y=e^{-x}$

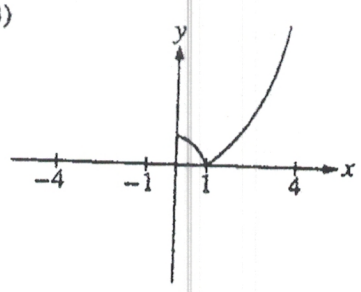


93 40. The graph of $y = f(x)$ is shown in the figure above. Which of the following could be the graph of $y = f(|x|)$?

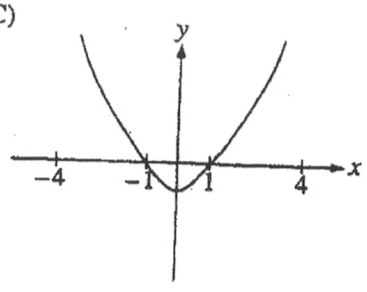
(A)



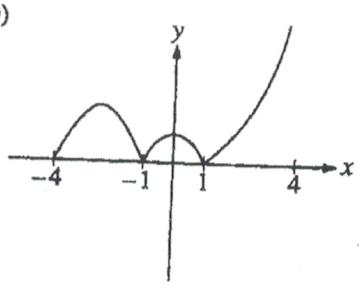
(B)



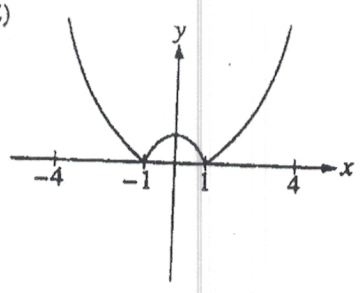
(C)



(D)



(E)



93 5. If the function f is continuous for all real numbers and if $f(x) = \dfrac{x^2 - 4}{x + 2}$ when $x \neq -2$, then $f(-2) =$

(A) -4 (B) -2 (C) -1 (D) 0 (E) 2

85 29. Which of the following functions are continuous for all real numbers x ?

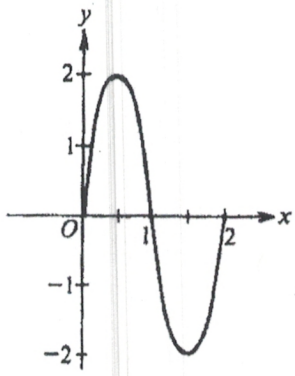
I. $y = x^{\frac{2}{3}}$

II. $y = e^x$

III. $y = \tan x$

(A) None (B) I only (C) II only (D) I and II (E) I and III

2



85 35. The figure above shows the graph of a sine function for one complete period. Which of the following is an equation for the graph?

(A) $y = 2\sin\left(\frac{\pi}{2}x\right)$ (B) $y = \sin(\pi x)$ (C) $y = 2\sin(2x)$

(D) $y = 2\sin(\pi x)$ (E) $y = \sin(2x)$

88 22. If $\ln x - \ln\left(\frac{1}{x}\right) = 2$, then $x =$

(A) $\frac{1}{e^2}$ (B) $\frac{1}{e}$ (C) e (D) $2e$ (E) e^2

93 13. The fundamental period of $2\cos(3x)$ is

(A) $\frac{2\pi}{3}$ (B) 2π (C) 6π (D) 2 (E) 3

69 AB 12. If $f(x) = \frac{4}{x-1}$ and $g(x) = 2x$, then the solution set of $f(g(x)) = g(f(x))$ is

(A) $\left\{\frac{1}{3}\right\}$ (B) $\{2\}$ (C) $\{3\}$ (D) $\{-1, 2\}$ (E) $\left\{\frac{1}{3}, 2\right\}$

3

Chapter 1 Review/Test Problems

1. For each polynomial function given here, determine:
   i) how many zeros does the function have?
   ii) to the nearest tenth, what is the largest zero?

   a) $f(x) = 3x^4 + 4x^3 - x + 0.05$
   b) $g(x) = -0.02x^4 + 0.96x^3 + 2.02x^2 - 0.98x - 1$

2. Find an equation of the line passing through the point A(4, –7) and perpendicular to the line through the points B(–1, –2) and C(5, 8).

3. An experiment yields the data shown in the table below:

t (hours)	0	1	2	3	4	5
p (pressure)	6	6.37	6.75	7.16	7.58	8.02

   a) Explain how you know that this data is not linear.
   b) Use your calculator to graph a scatter-plot of the data. What are your window dimensions?
   c) Write the equation of the linear regression function that might be used to model data.
   d) Using your linear model function, estimate the pressure when $t = 8.5$ hours.

4. a) If $f(x) = \sqrt{x^2 + 8x}$, find the domain of function f. Now write a formula for the function whose graph is obtained by:
   b) reflecting the graph of f across the y-axis;
   c) shifting the original graph of f to the left 3 units, flipping the graph over the x-axis, and then shifting it up 4 units.

5. A function f is given by the formula $f(x) = \dfrac{R}{x + K}$, where R and K are constants. If the points A(3, 2) and B(5, 1) both lie on the graph of f, determine the numbers R and K **exactly.**

6. "We left Andover at 5 o'clock, headed for Concord, a distance of sixty miles. Due to an accident on the interstate highway, traffic was creeping along for the first hour and a half. By 6:30 we were only twenty miles from Andover. After that we were able to speed up gradually and finally reached Concord at 7:30." Draw a possible graph to represent the distance (in miles from Andover) as a function of time (in hours since 5 o'clock) as described in the above quote. Explain the important features of your graph, including comments on its concavity.

7. If f is a polynomial function of degree 3 with the numbers 4, 5, and –6 as zeros, and if $f(1) = -210$, determine the formula for $f(x)$.

8. A rational function g has the lines $x = 2$ and $x = -2$ as vertical asymptotes, the line $y = 4$ as a horizontal asymptote, and the numbers 3 and 1 as zeros. Find a formula for $g(x)$.

9. Given the function $h(x) = \dfrac{3x}{x+5}$:

a) Explain how you know that h has an inverse function.

b) Find a formula for $h^{-1}(x)$.

10. The half-life of a toxic substance is 11,250 years.

a) If 153 gm of the substance is present now, write a formula for the function that gives the amount $A(t)$ that will be present t years from now.

b) When will only 0.1 gm of this substance remain?

11. a) Solve for x: $\log_2(3x-1) \le 4$

b) If $f(x) = \ln(x^2 + 4x)$, write a formula for the function g whose graph is obtained by reflecting the graph of f across the y-axis and then shifting it up 3 units and to the left 7 units.

12. Label each statement **TRUE** or **FALSE**.

a) $10^{\log(5+x^2)} = 5 + x^2$

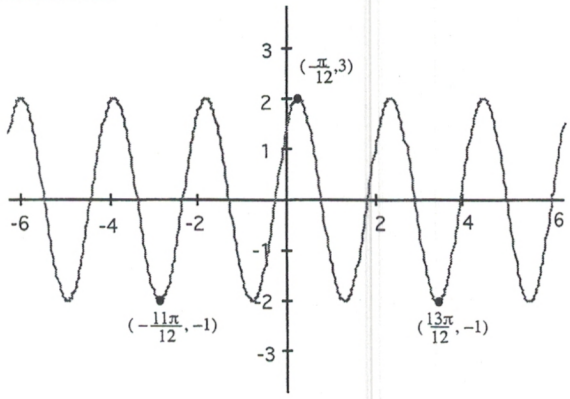
b) $\ln(A + B) = (\ln A)(\ln B)$

c) $\ln(\dfrac{A}{B}) = \ln(A) + \ln(\dfrac{1}{B})$

d) $e^{-\ln(C)} = -C$

e) $(\log_b(x))^y = y\log_b(x)$

13. Determine a possible formula for the function whose graph is shown below. Explain how your reach your conclusion.



14. Label each statement **TRUE** or **FALSE**.

a) $\sin(x^2) + \cos(x^2) = 1$ b) $\sin^2(x) + \cos^2(x) = 2$ c) $\cos(x + y) = \cos x + \cos y$

d) $\tan(2x) = 2\tan x$ e) $\sec x = \dfrac{1}{\sin x}$

15. In 1951, the population of India was 357 million people. By 1981 it had grown to 684 million. If the population is growing exponentially, when (in what month of what year) will the population reach 1 billion people?

16. Solve for x: $5 + \ln x = \dfrac{14}{\ln x}$

17. Dave invests \$100 at 8% interest per year. How much does Dave have after 6 years if the interest is

a) compounded annually; b) compounded quarterly; c) compounded continuously.

18. The population of New Hampshire was 1 million in 1990. It doubles every 25 years. Estimate the population in 1996.

19. Describe in words the graph of $y = 2^x$. (Hint: Talk about intercepts, asymptotes, increasing/decreasing, concavity, etc.)

20. For the following functions, write down an inverse function and verify that your answer is correct.

a) $f(x) = \dfrac{x+3}{3}$ b) $f(x) = \sqrt[3]{x-1}$ c) $f(x) = \dfrac{1}{x}$

21. Let $f(x) = \dfrac{ax+b}{cx-a}$, where a, b, and c are positive constants. Show that f is one-to-one and f is its own inverse function.

22. Label each statement **TRUE** or **FALSE**.

a) The sum of two one-to-one functions is one-to-one.

b) The product of two one-to-one functions is one-to-one.

c) If f is a one-to-one function and k is a real number (constant), then the function $g(x) = k \cdot f(x)$ is one-to-one.

23. Solve **exactly** for x: $\log_2(6-x) + \log_2(2-x) = 5$

24. Solve **exactly** for x: $x^{\ln x} = e^{100}$

25. Let $f(x) = \dfrac{x}{2x-3}$ and $h(x) = \dfrac{2}{x-1}$.

a) Determine a formula for the inverse function, $f^{-1}(x)$.

b) Write a simplified formula for the composite function $[h \circ f](x)$.

c) Determine the domain of $h \circ f$.

26. Suppose f and g are inverses of each other. What is true about their composition(s)?

27. Suppose $f(x) = x^2 - 1$ and $g(x) = \sin x$. Find

$$(f \circ g)(x) \ , \ (g \circ f)(x) \ , \ h(x) = [(f \circ g) \circ (g \circ f)](x) \ .$$

28. Suppose $a, b, c, d \geq 0$ and $y = a\sin(bx + c) + d$.

a) What is the maximum value of y ? b) What is the minimum value of y ?

c) What is the period of the function? d) Give two x-values for which $y = d$.

Answers Chapter 1 Review/Test Problems

1. a) i) two ii) 0.4 b) i) four ii) 50.0 2. $y = -0.6x - 4.6$

3. a) constant increases in x do not yield constant increases in y

   b) Zoom9 [–.5, 5.5] x [5.6566, 8.3634] c) $y = 0.404x + 5.97$ d) 9.4

4. a) Dom (f) = $(-\infty, -8]$ and $[0, \infty)$ b) $g(x) = \sqrt{x^2 - 8x}$ c) $h(x) = 4 - \sqrt{x^2 + 14x + 33}$

5. $R = 4, K = -1$

7. $f(x) = -2.5(x - 4)(x - 5)(x + 6)$ 8. $g(x) = \dfrac{4(x-3)(x-1)}{(x-2)(x+2)}$

9. a) graph of h passes horizontal line test b) $h^{-1}(x) = \dfrac{5x}{3-x}$

10. a) $A(t) = 153(0.99993838881)^t$ b) 119,017.3 years from now.

11. a) Solution set = $(\frac{1}{3}, \frac{17}{3}]$ b) $g(x) = 3 + \ln[(x+7)^2 - 4(x+7)]$

12. a) True b) False c) True d) False e) False

13. $f(x) = 1 + 2\cos(3x - \frac{\pi}{4})$

14. a) False b) False c) False d) False e) False

15. August 1998 16. $x = e^2$ or $x = e^{-7}$

17. a) \$158.69 b) \$160.84 c) \$161. 61 18. 1, 180, 992 people

19. The graph of $y = 2^x$ is always increasing and concave up. The graph is asymptotic to the left side of the x-axis, and has the point (0, 1) as its only axis-intercept.

20. a) $f^{-1}(x) = 3x - 3$ b) $f^{-1}(x) = x^3 + 1$ c) $f^{-1}(x) = \dfrac{1}{x}$

21. If $f(r) = f(s)$ then $\dfrac{ar+b}{cr-a} = \dfrac{as+b}{cs-a}$. Thus, $(ar + b)(cs - a) = (cr - a)(as + b)$ and

$$acrs + bcs - a^2r - ab = acrs - a^2s + bcr - ab,$$

$$(bc + a^2)\, s = (bc + a^2)\, r.$$

Since $bc + a^2 > 0$, it follows that $r = s$, so f is one-to-one.

22. a) False b) False c) False (unless $k \neq 0$)

23. $x = -2$ 24. $x = e^{10}$ or $x = e^{-10}$

25. a) $f^{-1}(x) = \dfrac{3x}{2x-1}$ b) $[h \circ f](x) = \dfrac{4x-6}{3-x}$ c) $R - \{3, 1.5\}$

26. $f \circ g = g \circ f = I$, where I is the identity function.

27. $[f \circ g](x) = \sin^2 x - 1$; $[g \circ f](x) = \sin(x^2 - 1)$; $h(x) = \sin^2(\sin(x^2 - 1)) - 1$

28. a) $a + d$ b) $-a + d$ c) $2\pi/b$ d) $-\dfrac{c}{b}$ or $\dfrac{\pi - c}{b}$